

# Numerical analysis of regenerators of free-piston type Stirling engines using Lagrangian formulation

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## Abstract

The circular duct between the cylinder and displacer serves as a regenerator in free-piston Stirling engines. The cylinder wall is fixed and the displacer wall is in reciprocating motion during the steady operation of the engine. The basic equations of the working fluid and regenerative duct are derived using the Lagrangian method in terms of the displacement of the displacer, so that time does not appear in the equations. A relation is derived between the cylinder and displacer wall temperatures to obtain the initial wall temperature distributions. A computer program is written in FORTRAN and the governing equations, which include the pressure fluctuations due to the flow reversals, are solved numerically using a finite difference method. The results and discussion are presented. © 2002 Elsevier Science Ltd and IIR. All rights reserved.

**Keywords:** Thermodynamic cycle; Stirling; Engine; Free piston; Regenerator; Operation; Calculation

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## Analyse numérique des régénérateurs des moteurs Stirling à l'aide de la méthode lagrangienne

**Mots clés :** Cycle thermodynamique ; Stirling ; Moteur ; Piston libre ; Régénérateur ; Fonctionnement ; Calcul

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### 1. Introduction

In Stirling-cycle machines the working fluid is alternatively heated and cooled, and the resulting pressure changes in the system perform useful work through volume changes. Beale's free-piston Stirling engine and Harwell's termo-mechanical generator have a duct between the cylinder and displacer and it serves as a regenerator [1,2]. While the cylinder wall is fixed, the displacer wall is in reciprocating motion during the operation of the machine. There are many investigations on regenerators, but in most cases only their applications to the gas turbines or cryogenerators are considered. For

the rapidly cycled regenerator of the Stirling-cycle machine, pressure fluctuations with time are important. The rapidly cycled regenerator theory allows for the effects of pressure fluctuations with time by inserting a term into the gas energy equation to account for the change in internal energy due to pressure pulsation. It is essential to know the heat transfer coefficient in order to predict the effectiveness of the regenerator.

In the literature of the theory of rapidly cycled regenerators, there are few solutions obtained by approximate methods. Assuming a square-wave variation with time for the mass flowrate and sawtooth variation with time for pressure, Rea and Smith [3] obtained parametric solutions for the effectiveness of the regenerator. Extending their theory further, Qvale and Smith [4] assumed sinusoidal pressure variation and mass flowrate with a phase angle between them and obtained an approximate solution for the thermal performance of a

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### Nomenclature

|        |                                                                                              |
|--------|----------------------------------------------------------------------------------------------|
| $A$    | Heat transfer area ( $\text{m}^2$ )                                                          |
| $A_a$  | Cross sectional area of the regenerative duct ( $\text{m}^2$ )                               |
| $b$    | Ratio of heat transfer coefficients ( $= h_{sd}d_d/h_{sy}d_y$ )                              |
| $c$    | Specific heat of the solid ( $\text{J kg}^{-1} \text{K}^{-1}$ )                              |
| $c_p$  | Specific heat at constant pressure ( $\text{J kg}^{-1} \text{K}^{-1}$ )                      |
| $c_v$  | Specific heat at constant volume ( $\text{J kg}^{-1} \text{K}^{-1}$ )                        |
| $d$    | Diameter (m)                                                                                 |
| $f_c$  | Convergence factor                                                                           |
| $h$    | Heat transfer coefficient ( $\text{W m}^{-2} \text{K}^{-1}$ )                                |
| $h_s$  | Displacement heat transfer coefficient ( $\text{J m}^{-3} \text{K}^{-1}$ )                   |
| $H$    | Enthalpy (J)                                                                                 |
| $L$    | Length of the duct (m)                                                                       |
| $m_e$  | Variation of mass with displacement of displacer at entry to the duct ( $\text{kg m}^{-1}$ ) |
| $M$    | Mass (kg)                                                                                    |
| $N$    | Velocity ratio ( $= m_e RT/A_a P$ )                                                          |
| $P$    | Pressure (Pa)                                                                                |
| $Q$    | Amount of heat transfer (J)                                                                  |
| $q_s$  | Amount of heat transfer per unit displacement of the displacer ( $\text{J m}^{-1}$ )         |
| $R$    | Gas constant ( $\text{J kg}^{-1} \text{K}^{-1}$ )                                            |
| $S$    | Amplitude of displacement movement (mm)                                                      |
| $T$    | Absolute temperature (K)                                                                     |
| $T_o$  | Inlet temperature to expansion volume (K)                                                    |
| $T'_o$ | Inlet temperature to compression volume (K)                                                  |

|            |                                                                                  |
|------------|----------------------------------------------------------------------------------|
| $u_d$      | Velocity of the displacer ( $\text{m s}^{-1}$ )                                  |
| $u$        | Velocity of the gas ( $\text{ms}^{-1}$ )                                         |
| $V$        | Volume ( $\text{m}^3$ )                                                          |
| $V_S$      | Swept volume ( $\text{m}^3$ )                                                    |
| $W$        | Amount of work done (J)                                                          |
| $x$        | Coordinate of a point on the displacer wall from the center of the cylinder (mm) |
| $y$        | Displacement of the displacer (mm)                                               |
| $y_e$      | Entry displacement of displacer for a gas element into annular duct (mm)         |
| $z$        | Coordinate of a point on cylinder wall from the center of cylinder (mm)          |
| $\eta$     | Effectiveness of the regenerator                                                 |
| $(1-\eta)$ | Ineffectiveness of the regenerator                                               |
| $\varphi$  | Phase angle (rad)                                                                |
| $\rho$     | Density ( $\text{kg m}^{-3}$ )                                                   |

### Subscripts

|    |                                        |
|----|----------------------------------------|
| a  | Regenerative duct                      |
| A  | Amplitude                              |
| C  | Compression volume                     |
| d  | Displacer wall                         |
| e  | At entry to the duct                   |
| H  | Expansion volume                       |
| id | Ideal                                  |
| in | Initial                                |
| M  | Mean                                   |
| s  | Per unit displacement of the displacer |
| t  | Total                                  |
| y  | Cylinder wall                          |

Stirling-engine regenerator. Kim [5] obtained a closed form solution for the amplitude of time-varying instantaneous pressure drops by using a sinusoidal pressure variation and mass flowrate.

The most comprehensive approach, known as nodal analysis, involves fixed subdivisions of working-fluid space in the regenerator. For each subdivision the basic equations are solved repeatedly in Eulerian coordinates. Using this approach Urieli [6] solved the basic equations numerically for each subdivision by a forced-convergence technique. Tew et al. [7] analyzed the thermodynamic characteristics of a rhombic-drive and a free-piston Stirling-cycle engine using nodal analysis. Urieli and Kushnir [8] showed that this analysis can be utilized in order to evaluate the various practical effects of non-ideal regenerators, heat exchangers, including heat transfer and pressure losses. Organ [9] made a nodal analysis of the Stirling-cycle machine and solved the basic equations for each subdivision to calculate pressure and gas velocity.

Rix [10] gave a thermodynamic simulation of a Stirling-cycle machine by using a Lagrangian formulation. In this approach the basic equations are formulated for

a section of the working fluid by using the gas density, velocity and temperature as variables, and cubic spline function is used to represent the matrix temperature distribution in the regenerator model.

## 2. Method of analysis

In the present analysis using a Lagrangian method, the governing equations of the regenerative duct are derived in terms of the displacement of the displacer. The equations, which include the pressure fluctuations due to flow reversals and longitudinal conduction are solved numerically by a digital computer using a finite difference method. The application of this method to the matrix regenerators of the Stirling-cycle machines is given in the previous studies [11,12].

The amount of convective heat transfer to or from a gas element based on the displacement of the displacer, is defined as

$$q_s = h_s A \Delta T \quad (1)$$

where  $h_s$  is the amount of heat transfer per unit area and displacement of the displacer. It is called the displacement heat transfer coefficient in the present analysis and is given by

$$h_s = h/u_d \quad (2)$$

The ineffectiveness of the regenerator for the compression half-cycle is defined as

$$(1 - \eta)_C = \frac{T'_0 - T_C}{T_H - T_C} \quad (3)$$

Similarly, for the expansion half-cycle is

$$(1 - \eta)_H = \frac{T_H - T_0}{T_H - T_C} \quad (4)$$

As an approximation the pressure of the system assumed to be a linear function of the displacement of the displacer and given as

$$P = P_M + \frac{2y}{S} P_A \quad (5)$$

Similarly, the masses of the gas in the expansion and compression volumes are respectively

$$M_H = M_{MH} - M_{AH} \frac{2y}{S} \quad (6)$$

and

$$M_C = M_{MC} + M_{AC} \frac{2y}{S} \quad (7)$$

The following additional assumptions are made in order to simplify the analysis:

- The displacement heat transfer coefficient is constant.
- The pressure drop across the regenerator is negligible.
- The longitudinal conduction within the cylinder and displacer walls and in the gas are negligible.
- The entry temperatures from the expansion and compression volumes,  $T_H$  and  $T_C$ , are constant.
- There is no radial temperature variation within the gas element, displacer or cylinder wall.
- The gas flow in the duct is one-dimensional and uniform at any section of the duct.
- The specific heats of the gas, displacer and cylinder walls are constant.
- The working gas can be treated as a perfect gas. The effect of the power piston is neglected.

The assumptions of a triangular waveform variation in the pressure and the mass of the expansion and com-

pression volumes with the displacement of the displacer simplify the analysis of the regenerator more than the sinusoidal variations that were assumed by Qvale and Smith [4] and Kim [5], and reflect the conditions in a Stirling engine better than the square-wave variation for the mass flowrate for pressure with time assumed by Rea and Smith [3]. Derivation of the basic equations with the Lagrangian formulation with respect to the displacement of the displacer also simplifies the analysis more than the approach used by Rix [10]. Another advantage of this approach is that the phase angle between the pressure and mass flow does not appear in the equations, and it simplifies the analysis of the complicated behavior of the machine. The coordinate system used throughout this analysis is shown in Fig. 1.

### 3. Basic equations

For the application of the Lagrangian method a gas element is considered at the entry to the duct, and observations are made on its state as it passes through the duct. The gas element is defined by two sections A and B, the coordinate of which are functions of two independent variables; the instantaneous displacement of the displacer,  $y$ , and the displacement,  $y_e$  at which the element under consideration entered the duct, as shown in Fig. 2.  $y$  and  $y_e$  are two variables defining the function  $z$  through a relationship of the form

$$z = z(y_e - y) \quad (8)$$

At a given displacement,  $y$ , the coordinates of the points A and B defining the element are

$$z_A = z(y_e, y) \quad (9)$$

and

$$z_B = z(y_e + dy_e, y). \quad (10)$$

After the displacement,  $dy$  the gas element, will occupy a different space B'A' in the duct. The coordinates of the points A' and B' will be

$$z_{A'} = z(y_e, y + dy), \quad (11)$$

and

$$z_{B'} = z(y_e + dy_e, y + dy). \quad (12)$$

By using the development of the Taylor series for  $z(y_e + dy_e, y)$ , neglecting higher order terms and Eq. (9), the length of the gas element at the displacement  $y$  is

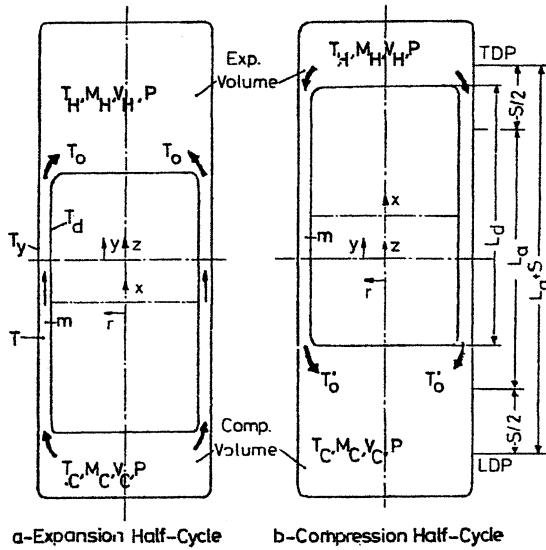


Fig. 1. The coordinate system.

$$z_B - z_A \cong \frac{\partial z}{\partial y_c} dy_c + \frac{1}{2} \frac{\partial^2 z}{\partial y_c^2} (dy_c)^2 \quad (13)$$

Owing to the interactions with the surroundings, the boundary of the gas element will change, but its mass will remain constant. By using a similar method for  $(z_{B'} - z_{A'})$ , the increase in the length of the gas element during the displacement,  $dy$  of the displacer can be written as

$$\delta z \cong -\frac{\partial^2 z}{\partial y \partial y_c} dy dy_c \quad (14)$$

For the gas element with a constant mass the continuity equation is written as

$$\frac{\partial z}{\partial y_c} = \frac{m_e RT}{A_a P} \quad (15)$$

where  $m_e$  is the mass flowrate of the gas in the duct with respect to  $y$  at  $y_c$ .

To derive the gas energy equation for the gas element the first law requires

$$\delta W_s = \delta Q_{sy} + \delta Q_{sd} + dE_s \quad (16)$$

During the compression half-cycle, the amount of heat transferred from the gas element to the cylinder and displacer walls is respectively.

$$\delta Q_{sy} = h_{sy} A_y (T - T_y) dy \quad (17)$$

and

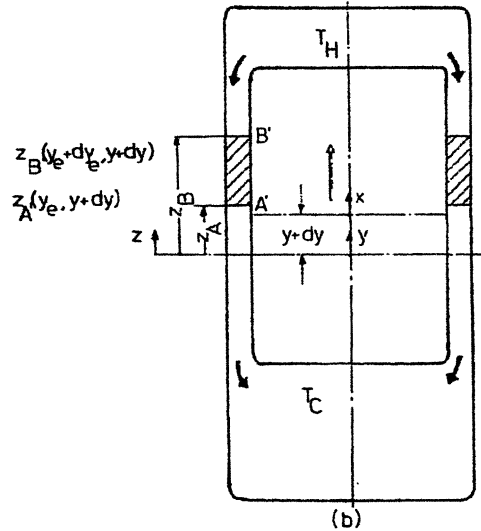
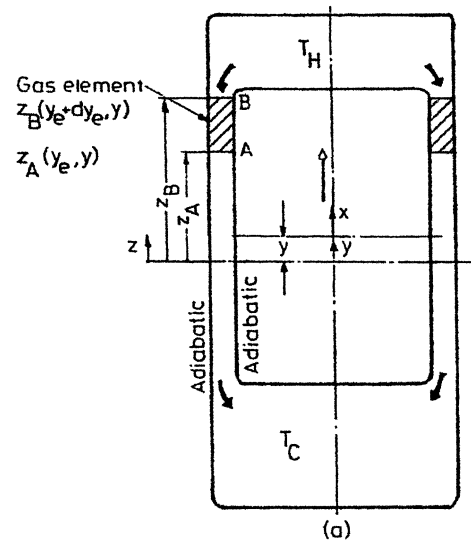


Fig. 2. Motion of gas element along the regenerator.

$$\delta Q_{sd} = h_{sd} A_d (T - T_d) dy \quad (18)$$

The work done by the gas element per unit displacement of the displacer

$$\delta W_s = P A_a \delta z \quad (19)$$

where  $\delta z$  is the increase in the length of the gas element during the displacement,  $dy$ , of the displacer and given by Eq. (14). Neglecting the changes in kinetic and potential energies, substitution of Eqs. (14), (17), (18) and (19) and the value of  $dE$  into Eq. (16) will eventually yield

$$A_a P \frac{\partial^2 z}{\partial y_e \partial y} + \pi d_y h_{sy} (T - T_y) \frac{\partial z}{\partial y_e} + \pi d_d h_{sd} (T - T_d) \frac{\partial z}{\partial y_e} + c_v m_e \frac{\partial T}{\partial y} = 0 \quad (20)$$

By using Eqs. (5), (7) and (15) with the perfect gas law, Eq. (20) will be

$$2A_a N \frac{P_A}{S} + \pi d_y N h_{sy} [(1+b)T(y) - T_{y(z)} - bT_d(x)] + 2c_p \frac{M_A}{S} \frac{\partial T(y)}{\partial y} = 0 \quad (21)$$

where

$$b = h_{sd} d_d / h_{sy} d_y \quad (22)$$

and

$$N = \frac{m_e R T}{A_a P} \quad (23)$$

From the energy balance for the cylinder wall, the cylinder wall energy equation including longitudinal conduction, is obtained as

$$\pi d_y h_{sy} (T - T_y) + \frac{k_y}{v_d} A_{cy} \frac{\partial^2 T_y}{\partial z^2} = \rho_y c_y A_{cy} \frac{\partial T_y}{\partial y} \quad (24)$$

Similarly the displacer wall energy equation is written as

$$\pi d_y h_{sd} (T - T_d) + \frac{k_d}{v_d} A_{cd} \frac{\partial^2 T_d}{\partial z^2} = \rho_d c_d A_{cd} \frac{\partial T_d}{\partial y} \quad (25)$$

For simplicity the length of the gas element is assumed constant and approximated by

$$z_2 - z_1 \cong -\frac{A_d}{A_a} dy_e \quad (26)$$

#### 4. Nondimensional form of the equations

The non-dimensional variables are chosen as:

$$\begin{aligned} \hat{z} &= z/L & \hat{x} &= x/L \\ \hat{y} &= y/S & \hat{y}_e &= y_e/S \\ \hat{T} &= T/T_C & \hat{T}_d &= T_d/T_C, \quad \hat{T}_y = T_y/T_C \\ \hat{P} &= P/P_M & \hat{P}_A &= P_A/P_M \end{aligned} \quad (27)$$

By using these non-dimensional variables, the non-dimensional form of the continuity equation is

$$\frac{\partial \hat{z}}{\partial \hat{y}_e} = A_1 \frac{\hat{T}}{\hat{P}} \quad (28)$$

The non-dimensional coefficient  $A_1$  in Eq. (28)

$$A_1 = \frac{m_e R S T_C}{A_a P_M L} \quad (29)$$

Similarly, from Eq. (21) the non-dimensional form of the gas energy equation can be written as

$$B_1 \hat{P}_A + B_2 (\hat{T} - \hat{T}_y) - B_3 (\hat{T} - \hat{T}_d) + B_4 \frac{\partial \hat{T}}{\partial \hat{y}} = 0 \quad (30)$$

The values of non-dimensional coefficients,  $B_1$ ,  $B_2$ ,  $B_3$  and  $B_4$  are

$$B_1 = N, \quad (31)$$

$$B_2 = \frac{\pi d_y h_{sy} S N T_C}{2 P_M A_a}, \quad (32)$$

$$B_3 = \frac{\pi d_d h_{sd} S N T_C}{2 P_M A_a}, \quad (33)$$

$$B_4 = \frac{c_p M_A T_C}{P_M A_a S} \quad (34)$$

From Eq. (24) the non-dimensional form of the cylinder wall energy equation

$$C_1 (\hat{T} - \hat{T}_y) + C_2 \frac{\partial^2 \hat{T}_y}{\partial \hat{z}^2} = \frac{\partial \hat{T}}{\partial \hat{y}} \quad (35)$$

The non-dimensional coefficients  $C_1$  and  $C_2$  are

$$C_1 = \frac{\pi d_y h_{sy} S}{\rho_y c_y A_a}, \quad (36)$$

$$C_2 = \frac{k_y}{\rho_y v_d c_y L_d} \quad (37)$$

From Eq. (25) the non-dimensional form of the displacer wall energy equation

$$D_1 (\hat{T} - \hat{T}_d) + D_2 \frac{\partial^2 \hat{T}_d}{\partial \hat{z}^2} = \frac{\partial \hat{T}}{\partial \hat{y}} \quad (38)$$

The non-dimensional coefficients  $D_1$  and  $D_2$  are

$$D_1 = \frac{\pi d_d h_{sd} S}{\rho_d c_d A_a}, \quad (39)$$

$$D_2 = \frac{k_d}{\rho_d v_d c_d L_d} \quad (40)$$

## 5. Numerical method

A simplified iterative numerical method is developed in order to solve the basic equations of the regenerator. This method can be stated in the following steps.

*Step 1.* A linear temperature distribution is assumed for the displacer wall;

$$\hat{T}_d(\hat{x}) = (T_H/T_C - 1)\hat{x} + (T_H/T_C + 1)/2 \quad (41)$$

Using Eq. (41), Eq. (A1) and (A2) are solved numerically and the initial temperature distributions of the displacer and cylinder walls,  $T_y(z)$  and  $T_d(x)$  are obtained.

By using these initial distributions of  $T_y(z)$  and  $T_d(x)$  the temperature distribution of the gas element is obtained for a fixed  $\hat{y}_e$  during the compression half cycle. When the first gas element enters the regenerator the initial conditions are;

$$\hat{y} = \hat{y}_e = -1/2, \quad (42)$$

$$\hat{z} = 1/2, \quad (43)$$

$$\hat{T} = T_H/T_C \quad (44)$$

The numerical integration of the gas energy equation begins with the above conditions. At the displacement of the displacer,  $\hat{y}$ , the conditions for a gas element with an entry displacement,  $\hat{y}_e$ , will be

$$\hat{y} = \hat{y}_e, \quad (45)$$

$$\hat{x} = 1/2 - \frac{(N+1)S}{L}(\hat{y} - \hat{y}_e) \quad (46)$$

$$\hat{z} = 1/2 - \frac{NS}{L}(\hat{y} - \hat{y}_e) + \frac{S}{L}\hat{y}_e \quad (47)$$

By using the outlet gas temperature from the regenerator, the ineffectiveness of the regenerator is calculated for each half cycle.

*Step 2.* The interval of  $\Delta\hat{z}$ , which will ensure the numerical stability, is determined using the inequalities obtained from Eqs. (35) and (38), and given by

$$C_1\Delta\hat{y}_e + \frac{2C_2\Delta\hat{y}_e}{(\Delta\hat{z})^2} \leq 1 \quad (48)$$

and

$$D_1\Delta\hat{y}_e + \frac{2D_2\Delta\hat{y}_e}{(\Delta\hat{z})^2} \leq 1 \quad (49)$$

By using the gas temperature distribution, the cylinder and displacer wall temperature distributions are

calculated from the cylinder and displacer wall energy equations.

*Step 3.* For all the  $\Delta\hat{y}_e$  intervals selected, *Steps 1* and *2* are repeated for the compression half-cycle.

*Step 4.* With the same intervals  $\Delta\hat{y}_e$  and  $\Delta\hat{z}$ , *Steps 1*, *2* and *3* are repeated for the expansion half-cycle.

*Step 5.* The net change of the cylinder and displacer wall temperatures over the compression and expansion half-cycles are written as

$$(\Delta\hat{T}_y)_C = (\hat{T}_y)_C + (\hat{T}_y)_i \quad (50)$$

$$(\Delta\hat{T}_y)_H = (\hat{T}_y)_H + (\hat{T}_y)_i \quad (51)$$

and

$$(\Delta\hat{T}_d)_C = (\hat{T}_d)_C + (\hat{T}_d)_i \quad (52)$$

$$(\Delta\hat{T}_d)_H = (\hat{T}_d)_H + (\hat{T}_d)_i \quad (53)$$

The net change of the cylinder and displacer wall temperatures over a cycle at each point are expressed as

$$\Delta\hat{T}_y = (\Delta T_y)_H + (\Delta T_y)_C \quad (54)$$

and

$$\Delta\hat{T}_d = (\Delta T_d)_H + (\Delta T_d)_C \quad (55)$$

Initial cylinder and displacer wall temperature distributions are corrected by these adjustments.

*Step 6.* By using the new temperature distribution of the cylinder and displacer wall,

*Steps 1–5* are repeated until they converge. The convergence is obtained when the heat transfer over a cycle to every point on the cylinder and displacer walls is negligible.

Fourteen points are selected on both the cylinder and displacer walls and the amount of heat transfer to and from the cylinder and displacer walls are calculated at these points for each half cycle. The heat capacity of the gas during a blow is much smaller than the heat capacity of the walls and the variation of the cylinder and displacer wall temperatures are very small. Therefore, the cylinder and displacer wall temperatures are corrected after each cycle using Eqs. (54) and (55).

Although the solution must go through a transient before reaching the overall steady-state, the convergence is accelerated by a convergence factor to minimize the computer time. The net change of the cylinder and displacer wall temperatures over a cycle given by Eqs. (54) and (55) are multiplied by this factor. This forced convergence technique is similar to that employed by Urieli [6] and Rix [10]. The method of the solution is given

schematically in Fig. 3 for the compression half-cycle. As shown in the figure for each entry displacement,  $y_e$ , a temperature distribution is obtained using the present method, and the gas element remained in the duct, is considered in the analysis. During the flow of gas elements in the duct, gas elements are in contact with the same heat transfer area on the displacer wall, but with different heat transfer area on the cylinder wall as shown in Fig. 3.

A computer program is written in FORTRAN and by using a finite-difference approach the governing equations are solved on a PC using this numerical method, details of which are given by Ataer [13].

## 6. Numerical results

In the present approach it is also essential to know heat transfer coefficients on the cylinder and displacer walls, which are assumed constant and calculated from the correlations given for periodically reversing flow. Air is used as the working fluid. The necessary geometrical and thermodynamic data for the numerical solution of the basic equations are given in Table 1. The interval of  $y_e$  is taken to be 10.0 mm and  $y$  to be 1.0 mm. The interval of  $z$  that will ensure numerical stability is calculated using Eqs. (48) and (49). The initial cylinder and displacer wall temperature distributions obtained from Eqs. (A1) and (A2) are given in Fig. 4.

For each fixed entry displacement,  $y_e$ , and with 10.0 mm intervals, a gas temperature distribution is obtained, as shown in Figs. 5–8. During the compression half cycle variation of the gas temperature along the displacer and cylinder walls are given in Figs. 5 and 6 respectively for

different entry displacements. Variation of gas temperature along the displacer wall, which is in reciprocating motion, is shown in Fig. 5. Variation of gas temperature along the cylinder wall, which is fixed, is shown in Fig. 6. The exit gas temperatures from regenerative duct are used to calculate the ineffectiveness of the regenerator. The temperature distribution of the gas that remains in the regenerative duct during the flow reversal is shown by  $y_e = 20$  mm in Figs. 5 and 6.

Similarly during the expansion half cycle variation of the gas temperature along the displacer and cylinder walls are given in Figs. 7 and 8, respectively. The dashed

Table 1

Input data for the numerical analysis of the regenerator

|                                                                                     |                          |
|-------------------------------------------------------------------------------------|--------------------------|
| Speed of cycling, $\omega$                                                          | 400 rpm                  |
| Stroke of the displacer, $S$                                                        | 60 mm                    |
| Compression volume temperature, $T_C$                                               | 387 K                    |
| Temperature difference between the expansion and compression volumes, $(T_H - T_C)$ | 150 K                    |
| Mean velocity of the displacer, $u_d$                                               | 0.71 m/s                 |
| Mean velocity of the air, $\bar{u}$                                                 | 9.24 m/s                 |
| Velocity ratio, $N$                                                                 | 14.29                    |
| Amplitude of the system pressure, $P_A$                                             | 15600 N/m <sup>2</sup>   |
| Amplitude of the mass variation, $M_{AC}$ , $M_{AH}$                                | 1.50 g                   |
| Displacement heat transfer coefficient on the cylinder wall, $h_{sy}$               | 383.9 J/m <sup>3</sup> K |
| Displacement heat transfer coefficient on the displacer wall, $h_{sd}$              | 414.3 J/m <sup>3</sup> K |

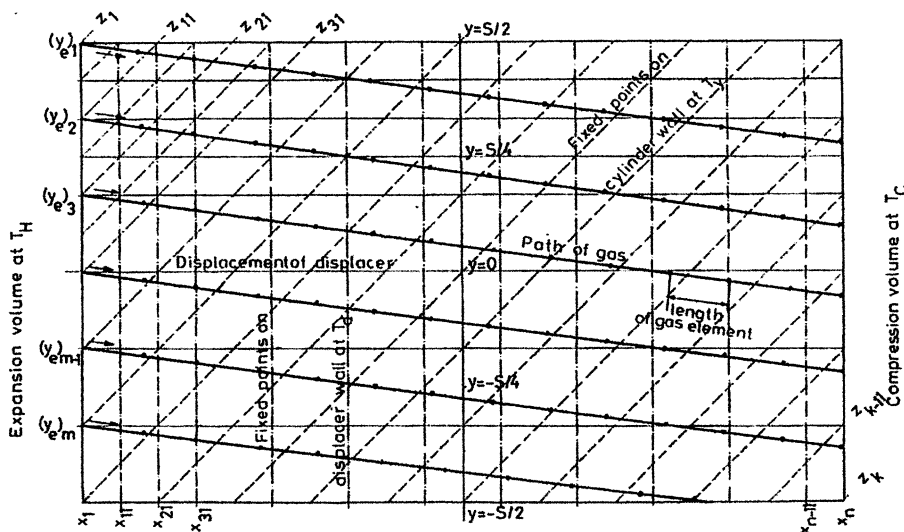


Fig. 3. Schematic diagram of the method of solution for the compression half-cycle.

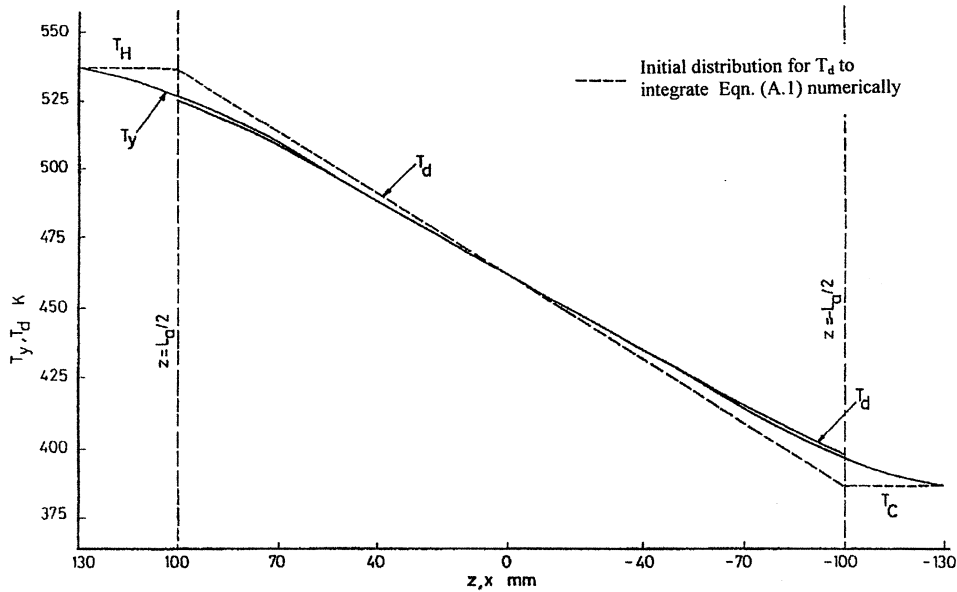


Fig. 4. Initial cylinder and displacer wall temperature distributions.

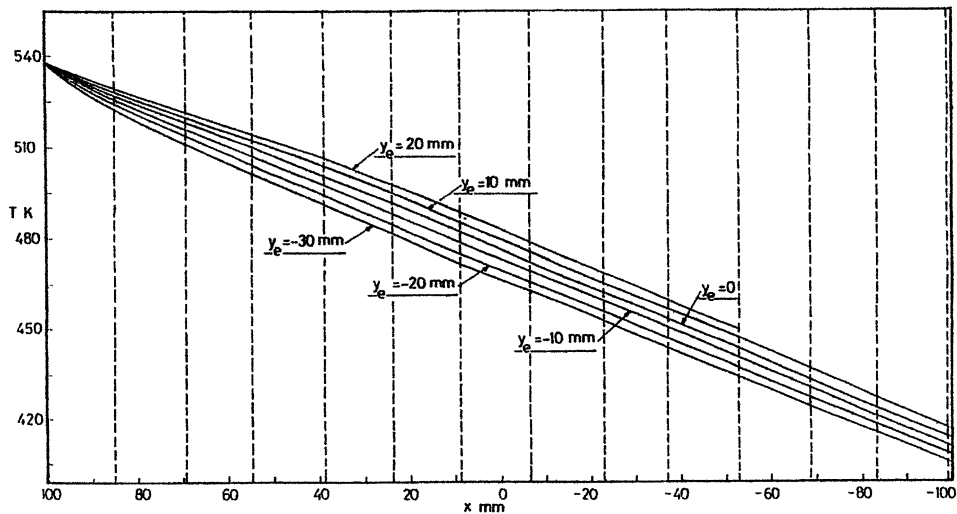


Fig. 5. Gas temperature distribution along the displacer wall during the compression half-cycle.

vertical lines in these figures show the location of the gas element in the regenerative duct.

When the steady operation of the engine is reached, the temperature distribution of the cylinder and displacer walls are obtained and given in Fig. 9. As seen in the figure the temperature distribution of the displacer wall slightly deviates from the cylinder wall temperature distribution at entry and exit of the regenerative duct.

As shown in these figures the present analysis computes the gas temperature distribution in the regenerative duct

for each entry displacement of the displacer, and also gives cylinder and displacer wall temperatures during steady operation of the engine. It simplifies the complicated behavior of the free-piston type Stirling engines.

By using Eqs. (3) and (4), the mean ineffectiveness of the regenerator over a cycle is defined as

$$(1 - \eta)_M = \frac{1}{2} \left( 1 + \frac{T'_{oM} - T_{oM}}{T_H - T_C} \right) \quad (56)$$



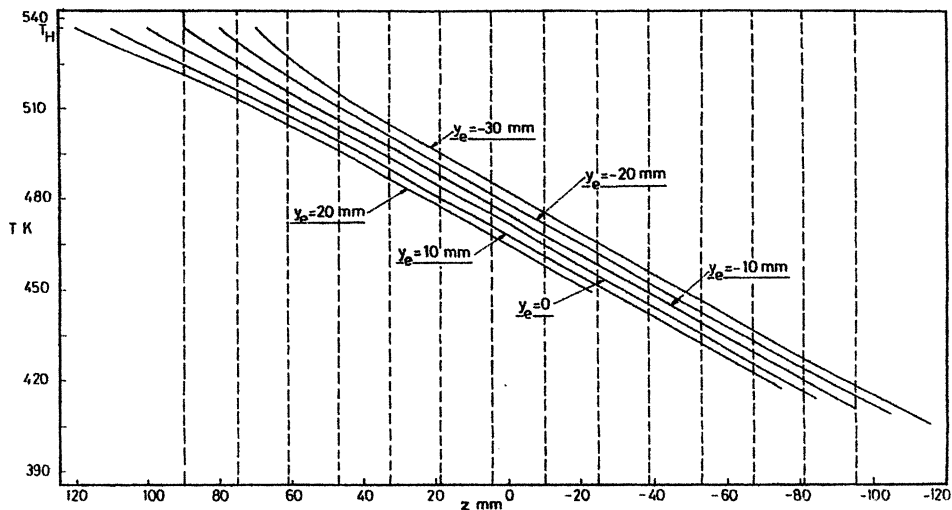


Fig. 6. Gas temperature distribution along the cylinder wall during the expansion half-cycle.

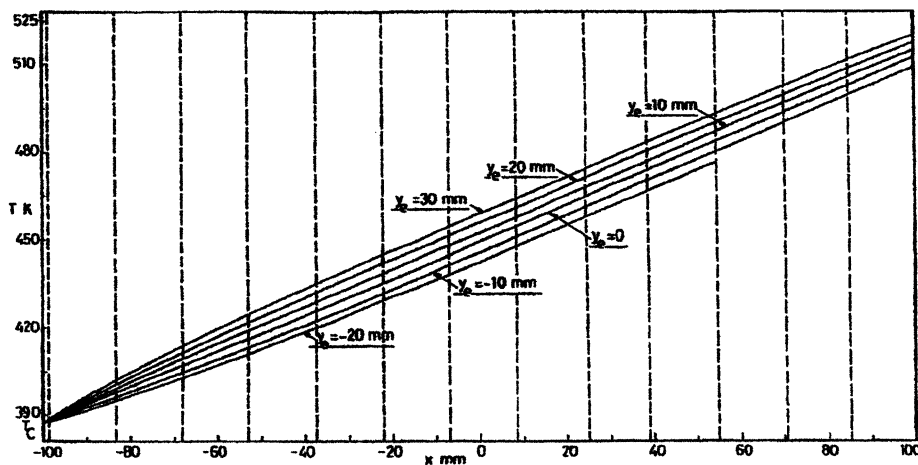


Fig. 7. Gas temperature distribution along the cylinder wall during the expansion half-cycle.

For the conditions given in Table 1 and using the data given for each fixed entry displacement by Figs. 5–8 the mean ineffectiveness of the regenerative duct over a cycle is calculated as 16.1%.

In the present analysis for each fixed entry displacement,  $y_e$ , the outlet temperature from the regenerative duct and ineffectiveness of the regenerator are calculated. Examples of the outlet temperatures from the regenerative duct are given in Fig. 10, and using the outlet temperatures in Eqs. (3) and (4), variation of the ineffectiveness with the outlet temperatures for each half-cycle are also given in Fig. 10. The data obtained from

the present analysis simplifies the design of the Stirling-cycle machines.

To minimize the computer time the convergence is accelerated by a convergence factor. Effect of the convergence factors on the acceleration of the convergence is shown in Fig. 11. As shown in the figure the solution for  $f_c = 2$  converges at cycles higher than 16 cycles, but it converges in 10 cycles for  $f_c = 7$ . An envelope of convergence is drawn for the convergence curves as shown in Fig. 11.

The assumption of constant-displacement heat transfer coefficient can be expressed as

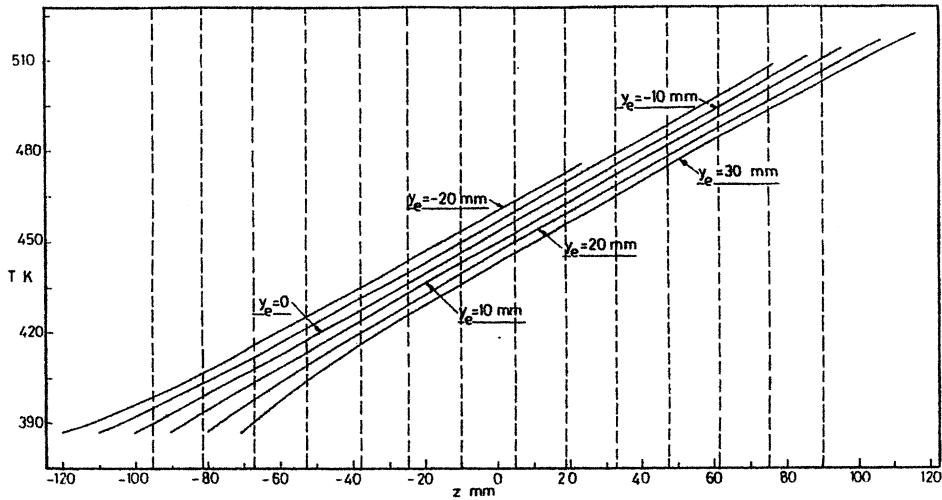


Fig. 8. Gas temperature distribution along the cylinder wall during the compression half-cycle.

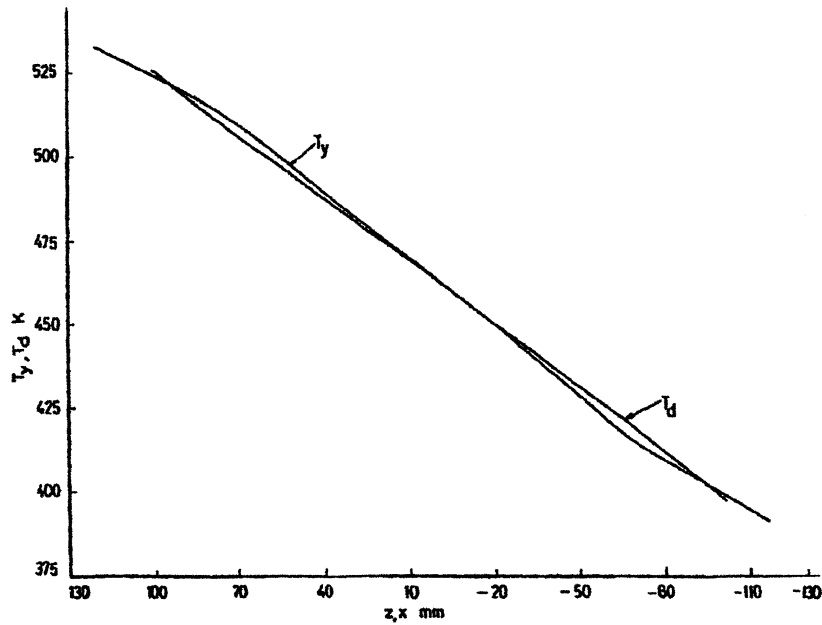


Fig. 9. The cylinder and displacer wall temperature distributions.

$$h_s = \left( \frac{h}{u_d} \right)_i \approx \frac{h}{u_d}, \quad (57)$$

and the relation between the gas and the displacer velocity approximated as

$$u = -\frac{A_d}{A_a} u_d \quad (58)$$

The velocity of the gas appears with the power of  $-0.2$  in the expression of displacement heat transfer coefficient and the error, caused by the assumption, is smaller than in the other theories based on the constant heat transfer coefficient

In the analysis the entry temperatures from the expansion and compression volumes,  $T_H$  and  $T_C$ , are assumed to be constant. Neglecting the effect of the

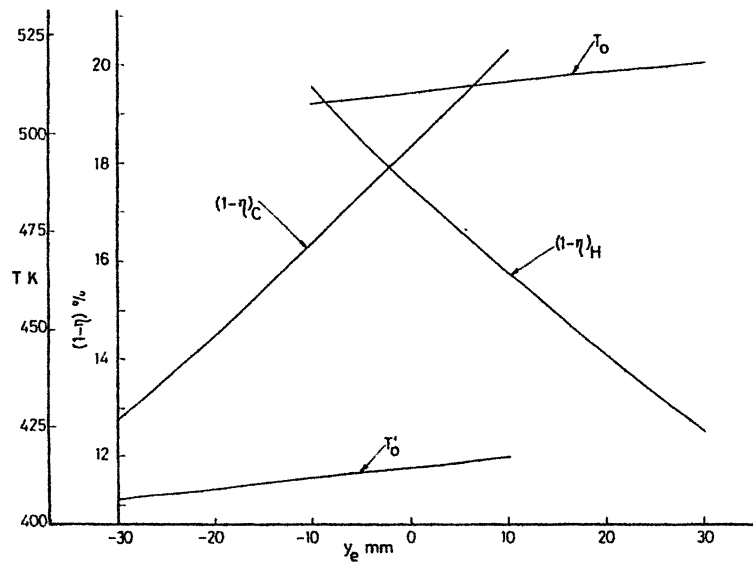


Fig. 10. Variation of ineffectiveness and outlet temperatures with the entry displacement of the displacer.

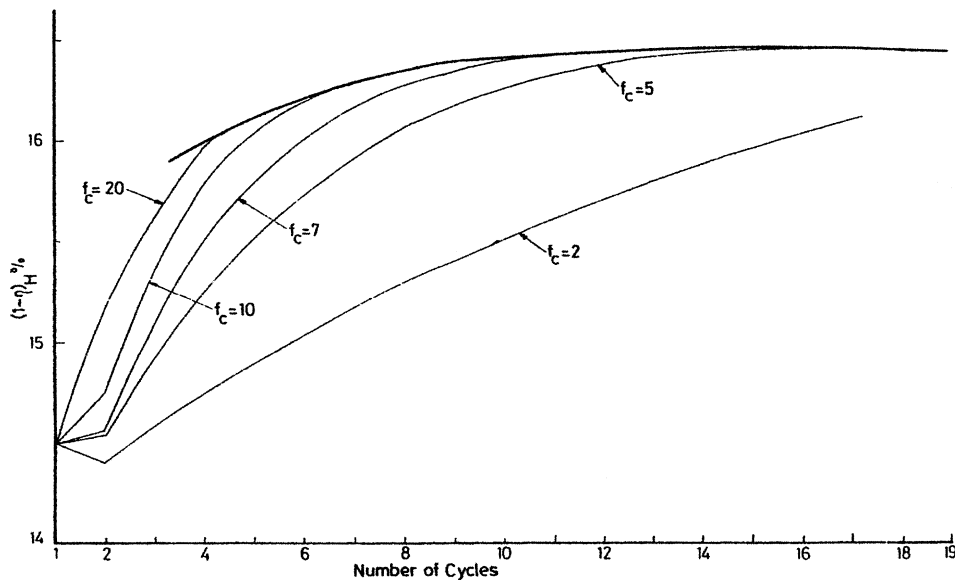


Fig. 11. Effect of different factors on the acceleration of the convergence.

piston, the volume variation of the expansion space with the displacement of the displacer is

$$V_H = V_{DH} + \left(1 - \frac{2y}{S}\right)V_S \quad (59)$$

where  $V_{DH}$  and  $V_S$  are the dead and swept volumes of the engine, respectively. Substituting Eqs. (5) and (6)

and (59) into the perfect gas law, and expanding the term  $[1 - (2y/S)M_{AM}/M_{MH}]$  as a binomial, it can be approximated as

$$T_H = C \left(1 - \frac{2y}{S}\right) \left[ \frac{P_{AM}}{P_M} + \frac{V_S}{V_{DH} + V_S} - \frac{M_{AM}}{M_{MH}} \right] \quad (60)$$

In Eq. (60), if the term in square brackets is equal to zero, then  $T_H$  will be constant. For the conditions given in Table 1 the magnitude of this term is equal to 0.2, which is quite small. It follows that whilst assuming linear variations for the system pressure and masses of the expansion and compression volumes, the temperatures of the expansion and compression spaces may be assumed constant.

## 7. Conclusions

A numerical method is demonstrated for obtaining an approximate solution for the heat transfer in the regenerative duct of free-piston Stirling engine with a reciprocating internal wall. The method appears to be more suitable for free-piston type Stirling engines than other theories which assume constant gas and constant metal temperatures at a location in the duct with time. Another advantage of the present method is the inclusion of the effects of the gas that remains in the regenerative duct during the flow reversals. The assumption of a constant-pressure process in the system at an instant and constant heat transfer coefficient, based on the displacement, introduces errors in the prediction of the ineffectiveness of the regenerator.

## Appendix

A simple method is developed to obtain a relation between the displacer and cylinder wall temperatures. The following assumptions are made to simplify this method:

- There is no gas flow in the duct.
- The longitudinal conduction is negligible in both the displacer and cylinder walls.
- Heat transfer between the displacer and cylinder takes place in proportion to the temperature difference.

The point Q on the cylinder wall with coordinate  $z$  is exposed to the temperature profile A–B in Fig. A1. The point R with coordinate  $z + dz$  is exposed to the temperature profile C–D. For there to be no net heat transfer from a point on the cylinder wall to the displacer, the average value of the temperature profile to which a point is exposed must be the same as its own temperature. By considering the shaded areas in Fig. A1 and their effect on the average height of the profiles to which the points Q and R exposed, the following equation is readily obtained

$$\frac{dT_y}{dz} = \left[ (T_d)_{(z+S/2)} - (T_d)_{(z+S/2)} \right] / S \quad (\text{A1})$$

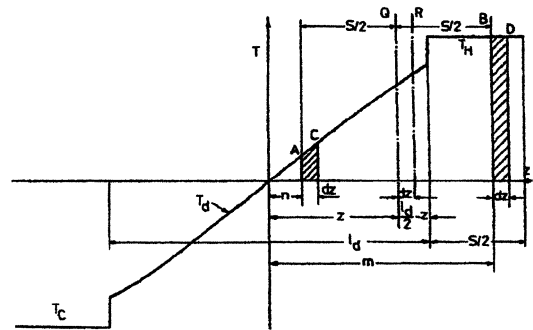


Fig. A1. Displacer-wall temperature distribution seen by points on the cylinder wall.

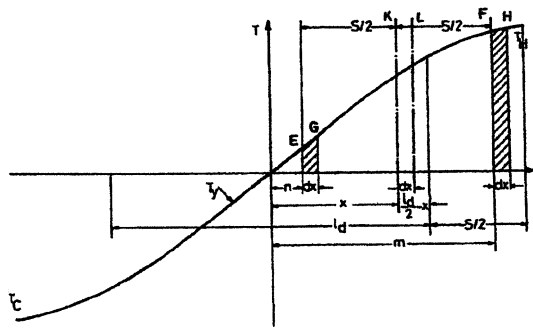


Fig. A2. Cylinder-wall temperature distribution seen by points on the displacer wall.

This equation is the condition for the quasi-static case, where the wall temperatures fluctuate periodically with small amplitude.

Similarly, using Fig. A2 the following equation can be obtained from the consideration of the temperature profile on the cylinder to which a point on the displacer is exposed.

$$\frac{dT_d}{dx} = \left[ (T_y)_{(xxS/2)} - (T_y)_{(x-S/2)} \right] / S \quad (\text{A2})$$

The initial temperature distributions of the displacer and cylinder walls are obtained using these relations derived between the cylinder and displacer wall temperatures.

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